QED

Queen's Economics Department Working Paper No. 1281

Higher Quality Exhaustible Resource Deposits Receiving Higher or Lower Resource Rents in a Simple Spatial Framework

John Hartwick
Queen's University

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

9-2011

Higher Quality Exhaustible Resource Deposits Receiving Higher or Lower Resource Rents in a Simple Spatial Framework

John M. Hartwick*(hartwick@econ.queensu.ca)
Queen's University, Kingston, Ontario

Abstract

Kolstad's (1994) model of intertemporal, competitive supply to a linear market from two distinct exhaustible resource deposits admits two different interior solutions – one with the low cost deposit "earning" the higher resource rent and the other with the low cost deposit "earning" the lower resource rent. This latter outcome turns on the initial size of the low cost deposit being significantly larger than the high cost deposit. We infer then that size can trump quality in the determination of the resource rent for a deposit, when geography is explicit.

- key words: exhaustible resource extraction, deposit quality, linear market
- highlights: lower cost deposits earning less rent>, resource extraction in a linear market>,evolution of market sizes>.
- JEL classification: D490; Q310; D210

1 Introduction

Kolstad's (1994) model of intertemporal, competitive supply to a linear market from two distinct exhaustible resource deposits admits two different interior solutions – one with the low cost deposit "earning" the higher resource rent and the other with the low cost deposit "earning" the lower resource rent. This latter outcome turns on the initial size of the low cost deposit being larger than the high cost deposit. We infer

*Gerard Gaudet provided comments to an earlier draft which put this research back on track. We are much indebted to him.

then that deposit size can trump *in situ* quality in the determination of the resource rent for a deposit. Such valuation scenarios have not appeared in aspatial Hotelling (1931) models with deposits of distinct qualities (eg. Herfindahl (1967)).[1] Our result does not turn on the relative accessibility of different deposit to markets. Accessibility would be a third factor, in addition to quality and deposit size, in the determination of resource rent and we have nothing to say about accessibility *per se* below. We proceed below to spell out the details of Kolstad's model and to work through in detail four complete numerical examples, the first a numerical rendering of the case Kolstad chose to analyze in his article (our Case A: the small, low-cost deposit is extracted from along with the larger high-cost deposit and the small, low-cost deposit is exhausted in a Phase I). Our other three cases are distinctly different versions of Kolstad's model. We distinguish then below among the Kolstad model, Kolstad's solution, our Case C solution, with the low cost deposit "earning" the lower resource rent, our Case B, with the low cost deposit "earning" the higher resource rent and our Case D, with a high-cost deposit of relatively small size.

A principal merit of Kolstad's spatial framework is that it allows for solutions with deposits of distinctly different qualities *in situ* supplying to a market at the same time. Morris Adelman among others has contended that Hotelling extraction theory was pretty well useless for explaining real-world oil extraction scenarios because such theory could not account for real-world supply flows emanating simultaneously from deposits of obviously distinct qualities (Adelman and Watkins (1992) and Cairns and Davis (2001)). The introduction of geography to Hotelling's framework,[2] along the lines of Kolstad (1994), turns out to be a sim-

[1] Nordhaus (1973) worked with a complicated Herfindahl model with deposits and demanders at different points on the surface of the earth. Buried in Nordhaus's empirical work should be a realization of the valuation phenomenon we are focusing on here.

[2] Gaudet, Moreaux, and Salant (2001) deal with exhaustible resource extraction

ple way to extend Hotelling extraction theory to a theory that deals satisfactorily with the Adelman problem. In addition, the introduction of explicit space allows one to move beyond the standard result that lowest-cost deposits "earn" highest resource rents.

The model has propective buyers of a unit of say oil located in uniform density on a linear market. Stocks are located at each end of the finite line, the right-hand one generally with lower, unchanging unit extraction cost. At each instant, resource rent at each supply point is rising at the rate of interest (Hotelling's zero profit intertemporal arbitrage condition). In Phase I, the market is split between competitive suppliers (think of each unit of stock owned by a distinct price-taking firm) by a delivered price, the same for each "supplier" (Cases B, C and D). This delivered price rises smoothly as time passes to an exogenously-set choke price, $\overline{p}$. At this date, Phase II opens with each "supplier" supplying to an own-market with an "edge" delivered-price at the choke price. A hole has opened in the "center" of the market and this hole grows steadily as time passes, with rents continuing to rise at the rate of interest. Resource rent at Right (R) rises to $\overline{p} - c^R$ at the instant of exhaustion and Left's rises to $\overline{p} - c^L$ at its instant of exhaustion, for c^i unit extraction cost for "supplier" i. For Case B (Right with unit costs distinctly lower and a stock size somewhat larger) we observe R with higher resource rent and an ever-contracting spatial market. For Case C (R and L with similar unit costs of extraction (R's lower) and R's stock size significantly larger than L's), we observe that R's resource rent is lower and her market-size is steadily expanding in Phase I. Market sizes for each "supplier" always contract in Phase II as time passes. Resource rent at each site rises smoothly at the rate of interest for both "suppliers" over both phases. There is no jump in rent or price at the date of the change in phases.

in a two dimensional space.

2 The Analysis

Our setting is supposed to be competitive and thus one should consider each unit of resource owned and supplied by a price-taking firm. We have used the word "supplier" above in the sense that supply is flowing out of a particular deposit but one should keep in mind that our setting is supposed to be truly competitive, with quantity supplied at a moment of time being "organized" by many small, price-taking firms.

Our linear market is of length G. Demanders are located on the line in uniform density. Each buyer buys one unit per unit time at the delivered price as long as this price is less than or equal to the cut-off or choke price, $\overline{p}$. We normalize so that distance is the same as quantity supplied.[3] Hence if $Q^L(t)$ is being supplied at an instant, the size of L's (left's) linear market is $Q^L(t)$ in units of distance. At each end, we have the left supplier indicated by L and right supplier indicated by R. Demanders buy a unit at delivered price, $p_m^i + \alpha u^i$ when located at distance u^i from supplier i. $i = L, R$. p_m^i is the mill price or price at the site of extraction. α is transportation cost per unit per unit distance. c^i is the unit extraction cost for supplier i. When extraction costs differ, we deal consistently with $c^R < c^L$. That is, R has the high quality stock *in situ*.

Benchmark results.

CHANGE IN STOCK SIZE:

Consider the symmetric case ($c^R = c^L$ and stock sizes (S_0^L and s_0^R) equal and selected so that each market size is initially at $G/2$). We have then split price, p^S equal to the choke price $\overline{p}$ initially for

$$p^S(0) = p_m^L(0) + \alpha\frac{G}{2} = p_m^R(0) + \alpha\frac{G}{2} = \overline{p},$$

for $p_m^L(0)$ and $p_m^R(0)$ mill prices at L and R respectively. The split-price

[3] If $u(t)$ is distance from the left supplier to its right hand market edge, then left's current demand is $\delta u(t)$ for δ uniform density of demanders on the line. We set $\delta = 1$ making current quantity demanded for the left supplier, $Q^L(t)$ equal to $\delta u(t)$.

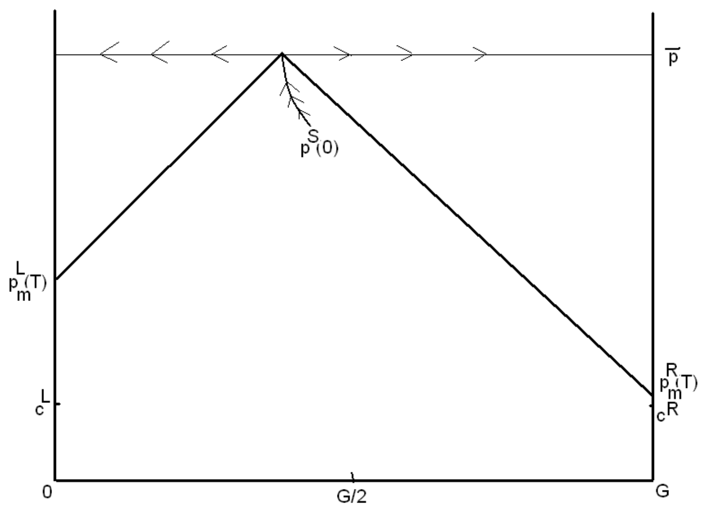

Figure 1: $p^S(0)$ moves up to $\overline{p}$ over Phase I. Each supplier extracts in isolation in Phase II.

is that which is dividing the whole market between the two suppliers at an instant. Beyond time zero, the resource rent at each site, $p_m^L(t) - c^L$ and $p_m^R(t) - c^R$ respectively, is rising at rate r and a hole is opening up in the center of the market as each supplier's market shrinks gradually, reaching zero at the instant rents reach $\overline{p} - c^L$ and $\overline{p} - c^R$ for L and R respectively. r is the market rate of interest and becomes the effective discount rate in our analysis.

We now add a small amount to R's initial stock and re-solve our supply problem. The new initial split-price will be below $\overline{p}$ and to the left of its counterpart above. R's initial rent will now be less than L's initial rent. The more abundant stock for R implies a lower value per unit. The new supply scenario will have two phases. Over each phase each supplier's rent will be rising at rate r and there will be no jumps. Over Phase I, the market will be fully supplied by the two "outputs" from the two suppliers and split-price will rise to $\overline{p}$ at time T. See Figure 1.

L's market size will be contracting over Phase I.[4] Phase II opens with a small hole opening in the "center" of the market and over Phase II each supplier's market size shrinks, reaching zero at the instant rents reach $\overline{p} - c^L$ and $\overline{p} - c^R$ for L and R respectively. Each initial stock is exhausted at the instant that respective rents reach $\overline{p} - c^L$ and $\overline{p} - c^R$. The duration of Phase II will in general differ for the two suppliers. At time zero, R's mill price will be lower and it will have the larger of the two market shares. As time passes its market share will be rising.

When one thinks about actually solving such a problem, one should work backwards in time from each supplier's end point. The determination of the split point in the market for the beginning of Phase II will be endogenous.

CHANGES IN EXTRACTION COSTS:

Our reference case is the symmetric one set out above. Now we consider c^R declining from its value when $c^R = c^L$. The new initial split-price will be below $\overline{p}$ and to the left of its counterpart above. R's initial rent will now be larger than L's initial rent. The higher quality for R's stock implies a higher value per unit. The new supply scenario will have two phases. Over each phase each supplier's rent will be rising at rate r and there will be no jumps. Over Phase I, the market will be fully supplied by the two "outputs" from the two suppliers and split-price will rise to $\overline{p}$ at time T. L's market size will be expanding, in this case, over Phase I. Phase II opens with a small hole opening in the "center" of the market and over Phase II each supplier's market size shrinks, reaching zero at the instant rents reach $\overline{p} - c^L$ and $\overline{p} - c^R$ for L and R respectively. Each initial stock is exhausted at the instant that respective rents reach $\overline{p} - c^L$ and $\overline{p} - c^R$. The duration of Phase II will in general differ for the two suppliers.

These are two basic comparative statics exercises that can guide our

[4] This follows from the fact that R's initial rent has become smaller than L's and each rent must rise at rate r over Phase I.

intuition for our following four cases.

3 Case A (Kolstad): Small High-quality Initial Stock

In Phase I, of length T, the full market is being supplied by both suppliers with the markets separated by split-price,

$$p^S(t) = p_m^L(t) + \alpha Q^L(t) = p_m^R(t) + \alpha[G - Q^L(t)].$$

At each date mill price $p_m^i = c^i + \lambda^i(t)$ for $\lambda^i(t)$ the current resource rent at the site of deposit i. Phase I ends with $Q^R(T) = S^R(T) = 0$ and $Q^L(T) = G$. There will be a delivered price at date T with $p(T) = c^L + \lambda^L(T) + \alpha G$ at the market edge at G.

Consider an example.

We assume $c^R = 1.0$, $c^L = 2.9$, $r = 0.1$, $\alpha = 0.1$ and $G = 40$. We proceed to fix L's stock size which she puts on the market in Phase I at 110 units and to solve for R's initial stock, S_0^R which solves Phase I correctly (L's initial market size grows over the interval to G).

We have then rents in Phase I

$$\begin{aligned} \lambda^L(t) &= [\overline{p} - \alpha Q_T^L - c^L]e^{-r[T-t]} \\ \text{and } \lambda^R(t) &= [\overline{p} - \alpha[G - Q_T^L] - c^R]e^{-r[T-t]}. \end{aligned}$$

We substitute these rents into the split-price relation to obtain

$$Q^L(t) = \frac{1}{2\alpha}\{[c^R + \alpha G - c^L]\{1 - e^{-r[T-t]}\} + 2\alpha Q_T^L e^{-r[T-t]}\}.$$

Recall that the value of Q_T^L is set at G. Hence we can solve for T in $S_0^L = 110 = \int_0^T Q^L(t)dt$. We integrate $\int_0^T Q^L(t)dt$ and obtain

$$S_0^L = \left[\frac{c^R + \alpha G - c^L}{2\alpha}\right] T - (1 - e^{-rT}) \left[\frac{c^R + \alpha G - c^L - 2\alpha Q_T^L}{2\alpha r}\right].$$

For $S_0^L = 110$,we obtain $T = 3.06323$. Since $TG = S_0^L + S_0^R$ and have solved for T, we have a value for R's total sales over Phase I, namely

$S_0^R = 12.5292$. Hence we have obtained S_0^R as a function of S_0^L. We obtain $Q^L(0) = 32.2164$, and with $Q_T^L = 40$, we have

$$\lambda^L(0) = [\overline{p} - \alpha Q_T^L - c^L]e^{-3.06323r} = 3.7544$$
$$\text{and } \lambda^R(0) = [\overline{p} - \alpha[G - Q_T^L] - c^R]e^{-3.06323r} = 8.0976.$$

The relatively large initial rent for R implies, given each rent rising at rate r, that R's market will be shrinking as time passes. Given the values for initial rents, we have the prices, $p^R(0) = 9.0976$ and $p^L(0) = 6.6544$. Beyond T, marking the end of Phase I, is Phase II with L supplying alone in the market, an amount $Q^L(t) = G$ at each instant until delivered price reaches $\overline{p}$. Phase II ends with L's delivered price reaching $\overline{p}$. Her rent is rising smoothly over Phase II at rate r. In Phase III, L's rent continues to rise at rate r and her market size is shrinking smoothly, with delivered price at her market edge being $\overline{p}$ at each instant. Phase III ends with $Q^L(t)$ and $S^L(t)$ each reaching zero.[5]

We solved our example by fixing L's amount of stock to be put on the market in Phase I at 110 units. A more satisfactory approach and more difficult to execute would have R and L's total stocks set initially and the supply paths worked out backwards from the final date of L's supply. In this case, L's amount of stock to be put on the market in Phase I would be endogenous. Such a somewhat more satisfactory solution would be qualitatively unchanged from ours above. Clearly our approach to solving has the merit of relative simplicity. We employ a variant of this approach in our analyses below.

Our example above is the same, in a qualitative sense, as the one reported by Kolstad (1994). It can be characterized as having the low-

[5]If L were alone in the market (R with initial stock of zero units) and we worked back from the end over two phases, we would run into the cookie-cutter problem: L's stock would not fit precisely in the time slots and its $Q^L(0)$ would not equal G in general. However, once we have R with some stock, we can, with reasonable parameters, eliminate the cookie-cutter problem.

cost supplier "move first" (here in fact simultaneous with the high-cost producer) and exhaust her relatively small stock (low-cost) in a Phase I. Each supplier's rent *in situ* is rising at rate r in Phase I and the phase ends with the low-cost supplier exhausting her initial stock just as the high-cost supplier is about to supply to all demanders in the linear market. Kolstad did not inquire about cases with each supplier with an initial stock of a similar size or about cases with the low-cost supplier having a much larger initial stock. These variants are not trivial extensions of our "base case" above. We turn to these novel variants.

4 Case B: Similar stock sizes, Distinctly different qualities

In Phase I, of length T, the full market is being supplied by both suppliers with the markets separated by split-price,

$$p^S(t) = p_m^L(t) + \alpha Q^L(t) = p_m^R(t) + \alpha[G - Q^L(t)].$$

At each date mill price $p_m^i = c^i + \lambda^i(t)$ for $\lambda^i(t)$ the current resource rent at the site of deposit i. Phase I ends with $p^S(T) = \overline{p}$. Hence over the interval 0 to T, the delivered split-price rises for each supplier to its terminal value, $\overline{p}$. The split in the market is defined by quantity $Q^L(t)$, current supply of L. The other portion of the market is then $G - Q^L(t)$ and is current output from R. At the end of this initial phase, we observe L supplying Q_T^L and R supplying $G - Q_T^L$. Over this first Phase we indicate S_0^L and S_0^R as total supply put on the market from L and R respectively. Total initial stocks are then K^L and K^R.

In Phase II, there are positive amounts of each stock remaining, namely $\widetilde{S}^L = K^L - S_0^L$ and $\widetilde{S}^R = K^R - S_0^R$. Beyond T, a hole opens smoothly in the "center" of the market ($Q^L(t)$ declines from Q_T^L and $Q^R(t)$ declines from $G - Q_T^L$) as each supplier has her market size shrink smoothly with delivered price at the spatial margin for each market remaining at $\overline{p}$. Rent at the supply point continues to rise at the rate of interest for each supplier. $\widetilde{S}^L$ and $\widetilde{S}^R$ each are exhausted at the moment that rent for L reaches $\overline{p}-c^L$ and rent for R reaches $\overline{p}-c^R$, respectively. The durations of exhaustion for $\widetilde{S}^L$ and $\widetilde{S}^R$ will be different in general. Hence it is correct to say that each supplier has a Phase II of different length.

An easy way to fill in details of our equilibrium is to first treat $\widetilde{S}^L$ as exogenous with $S_0^L = K^L - \widetilde{S}^L$ and $\widetilde{S}^R$ and S_0^R as endogenous. In this approach one has $\widetilde{S}^R$ and S_0^R functions of $\widetilde{S}^L$. (Later one obtains the complete solution by relaxing the assumption of $\widetilde{S}^L$ as exogenous. One then has each of $\widetilde{S}^R$, S_0^R and S_0^L as endogenous, with $\widetilde{S}^R + S_0^R = K^R$ and $\widetilde{S}^L + S_0^L = K^L$.)

Detailed analysis: We start in Phase II with $\widetilde{S}^L$ exogenous. We are solving back from end dates. Market size for L defined by $Q^L(t)$ satisfies at each instant

$$c^L + \lambda^L(t) + \alpha Q^L(t) = \overline{p}$$

with $\lambda^L(t) = [\overline{p}-c^L]e^{-r[\widetilde{T}^L-t]}$ for $\widetilde{T}^L$ the duration of Phase II for L. Hence $Q^L(t) = \frac{\overline{p}-c^L}{\alpha}\left[1-e^{-r[\widetilde{T}^L-t]}\right]$. Given $\int_0^{\widetilde{T}^L} Q^L(t)dt = \widetilde{S}^L$, we can solve for the value of $\widetilde{T}^L$ and the value of $Q^L(t)$ at the beginning of Phase II. Upon integrating $\int_0^{\widetilde{T}^L} Q^L(t)dt$, we obtain

$$\widetilde{S}^L = \frac{\overline{p}-c^L}{\alpha}\left[\widetilde{T}^L + \frac{1}{r}(e^{-r\widetilde{T}^L}-1)\right].$$

We proceed with a numerical example. Let $G = 40$, $\alpha = 0.1$, $c^R = 1$, $c^L = 2.9$, $\widetilde{S}^L = 24$, $\overline{p} = 12$, and $K^L = 31$.[6] Solving yields

$$\widetilde{T}^L = 2.38806$$

$$\text{and } Q_T^L = 19.33133.$$

[6]Observe that $c^R + \alpha G = 5$ and this exceeds $c^L = 2.9$. This is a condition which Kolstad imposes in order that the right-hand supplier not under-cut the left-hand supplier in terms of costs. More on this below.

This latter quantity we write as Q_T^L for T the length of Phase I. Our solved Q_T^L is about half the whole market. Q_T^L is defining the market split point at the end of Phase I.

With regard to R, we have her rent at the end of Phase I as

$$\lambda^R(T) = [\overline{p} - \alpha[G - Q_T^L] - c^R].$$

This rent must equal the discounted rent at the end of Phase II, namely $[\overline{p} - c^R]e^{-r\widetilde{T}^R}$ for $\widetilde{T}^R$ the length of Phase II for R. Given Q_T^L, we solve for $\widetilde{T}^R$in

$$\widetilde{T}^R = -\frac{1}{r}\ln\left\{\frac{[\overline{p} - \alpha[G - Q_T^L] - c^R]}{[\overline{p} - c^R]}\right\}.$$

We obtain $\widetilde{T}^R = 2.08128$ and

$$\widetilde{S}^R = \frac{\overline{p} - c^R}{\alpha}\left[\widetilde{T}^R + \frac{1}{r}[e^{-r\widetilde{T}^R} - 1]\right] = 22.2542.$$

$\widetilde{S}^R$ is the sum of sales for R in Phase II. This is a value for $\widetilde{S}^R$ as a function of $\widetilde{S}^L$, this latter exogenous for the moment.[7]

We turn to Phase I.

Over Phase I, the sum of sales from the two suppliers fills the market of length G at each instant. In Phase I, the markets are separated at distance $Q^L(t)$ at each moment by split-price,

$$p^S(t) = p_m^L(t) + \alpha Q^L(t) = p_m^R(t) + \alpha[G - Q^L(t)].$$

At each date mill price $p_m^i = c^i + \lambda^i(t)$ for $\lambda^i(t)$ the current resource rent. This rent is assumed to be rising at the rate of interest, r. We have then rents in Phase I

$$\lambda^L(t) = [\overline{p} - \alpha Q_T^L - c^L]e^{-r[T-t]}$$
$$\text{and } \lambda^R(t) = [\overline{p} - \alpha[G - Q_T^L] - c^R]e^{-r[T-t]}.$$

[7] $\widetilde{T}^R = 2.08128$ can be read as Phase II involving about 2 time periods, in discrete time, for R. R's supply in its second-to-last period is then about 40-19.33133 $\cong$ 20.7, leaving about 1.6 units to put on the market in its final period.

We substitute these rents into the "split price" relation to obtain

$$Q^L(t) = \frac{1}{2\alpha}\{[c^R + \alpha G - c^L]\{1 - e^{-r[T-t]}\} + 2\alpha Q_T^L e^{-r[T-t]}\}.$$

Recall that the value of Q_T^L has been solved for in Phase II, above. Hence we can solve for T in $S_0^L = K^L - \widetilde{S}^L = \int_0^T Q^L(t)dt$. We integrate $\int_0^T Q^L(t)dt$ and obtain

$$S_0^L = \left[\frac{c^R + \alpha G - c^L}{2\alpha}\right] T - (1 - e^{-rT}) \left[\frac{c^R + \alpha G - c^L - 2\alpha Q_T^L}{2\alpha r}\right].$$

For $S_0^L = 7$,we obtain $T = 0.36511$. Since $TG = S_0^L + S_0^R$ and have solved for T, we have a value for R's total sales over Phase I, namely $S_0^R = 7.6046$. Hence we have obtained S_0^R as a function of $\widetilde{S}^L$. Note then that $K^L = 24 + 7$ and $K^R = 22.2542 + 7.6046$. Hence total stocks put on the market differ by only about one unit for the two suppliers or the size of each deposit is about the same at time zero.

Initial rents work out to be

$$\begin{aligned} \lambda^L(0) &= [\overline{p} - \alpha Q_T^L - c^L]e^{-0.36511r} = 6.9099 \\ \text{and } \lambda^R(0) &= [\overline{p} - \alpha[G - Q_T^L] - c^R]e^{-0.36511r} = 8.6129 \end{aligned}$$

and the initial quantity extracted by L is 19.0147 and by R is (40-19.0147).[8]

Note that the point of market split in Phase I is moving TO THE RIGHT, from 19.0147 initially to 19.33133 at date T. This type of motion is a direct consequence of $\lambda^R(0) > \lambda^L(0)$ and subsequent rents moving up at $r\%$. (A simple sketch establishes this.)

There are no loose ends to our analysis above. However, matters were much simplified by our taking $\widetilde{S}^L$ and S_0^L as exogenous and S_0^R and $\widetilde{S}^R$ as endogenous. Conceptually, it is easy to now dispense with this

[8] We have verified that when these initial rents rise at r% over the combined time intervals for the two phases, we get the correct terminal rents.

crutch. Suppose now that we have $\widetilde{S}^L$ and S_0^L only to be positive and sum to 31, and S_0^R+ $\widetilde{S}^R$ equal to say 31, with each positive. 31 is close to the sum of S_0^R and $\widetilde{S}^R$ that we observed above (22.2542 + 7.6046). Hence one can envisage a brute for search over values of $\widetilde{S}^L$ near 24 (with $S_0^L + \widetilde{S}^L = 31$) that in fact brings the sum of S_0^R and $\widetilde{S}^R$, endogenous still, to the exogenously set value of 31. We would then have a solution devoid of the "pre-cooking" of the values of $\widetilde{S}^L$ and S_0^L.[9]

5 Case C: Similar Stock Qualities and Sizes

It turns out to be straightforward to change our parameters to obtain an "inverse" of the above case, one now with the low-cost supplier "pushing" the point of market split to the left and "earning" a LOWER RENT initially *in situ*. A parameter change that works is simply bringing the two extraction costs close ($c^L = 2.9$ and $c^R = 2.8$ and following the same solution steps as with Case B. This change leads to R putting a larger amount on the market in Phase I).[10]

With c^R equal now at 2.8 (up from 1.0), we can solve in this case for $\widetilde{T}^R$in $[\overline{p} - c^R]e^{-r\widetilde{T}^R} = [\overline{p} - \alpha[G - Q_T^L] - c^R]$, obtaining 2.5445. (Recall that $Q_T^L = 19.3313$.) This leads to a new value for $\widetilde{S}^R$, namely 27.41 (up from the earlier value of 22.2542). The new value for T solves out as 0.36205. R's total supply in Phase I comes out as 7.482 (=S_0^R). Hence K^L is the same at 40 units and K^R is now 27.41+7.482$\cong$ 35, up from 30 for Case B.

[9] It appears that it would not be difficult to specify parameters *a priori* that would be incompatible with an interior solution of the type we have spelled out. This is probably generic to equilibria with demand specifications with choke prices.

[10] Recall that above at the beginning of Phase II, we had

$$\lambda^L(T) = [\overline{p} - \alpha Q_T^L - c^L]$$
$$\text{and } \lambda^R(T) = [\overline{p} - \alpha[G - Q_T^L] - c^R].$$

Since each of these values gets discounted at the same rate and over the same interval in order over Phase I, we see that $\lambda^L(T) > \lambda^R(T)$ will lead to $\lambda^L(0) > \lambda^R(0)$. We solved for $Q_T^L = 19.33133$ above, less than half of market size at 40. Hence inspection of $\lambda^L(T)$ and $\lambda^R(T)$ above reveals that if c^R were only a small amount less than c^L, we would have $\lambda^L(T) > \lambda^R(T)$. This is our cue for re-solving.

And our choice of c^R gives us $\lambda^L(0) = 6.912$, a value GREATER THAN $\lambda^R(0) = 6.8795$. These values go along with L's market CONTRACTING over Phase I (from 19.3373 initially to 193313). We infer that we that a relative abundance effect has yielded our reversal of results in this case from results for Case B: here the similarity in extraction costs and larger initial stock for R has yielded a lower resource rent for R's deposit.[11]

Cases B and C illustrate the new economics in this contribution: in a spatial framework, the high quality stock can be "earning" a lower resource rent *in situ* because a relative stock size effect is trumping a relative quality effect.

6 Case D: Small Initial High-cost Stock

The idea here is that the low-cost stock should be extracted from first, a variant of the Herfindahl scenario. R should be the lone supplier over a positive interval, with a market size G, and then the high-cost stock should "enter". Then both exhaust in some kind of end scenario. Central is the idea that each supplier's rent should be rising over time at the rate of interest, ideally free of jumps that might induce suppliers to game possible jumps to the advantage of one or both. We consider then a Phase I with R alone with supply G at each instant. A Phase II opens with the high-cost supplier entering with an infinitesimal supply while the low-cost supplier supplies the remaining part of the market. Gradually over Phase II, the high-cost supplier's market expands while split-price rises to $\overline{p}$. This ends Phase II. Over Phase III, each supplier's market contracts smoothly to zero and each exhausts when rent reaches $\overline{p} - c^L$ for L and $\overline{p} - c^R$ for R. Note Phase III will in general be of a different duration for L and R. Our detailed solving turns up a variant of this solution, only slightly different.

[11] Clearly our initial rents for Case C differ by a small amount and the shift in the point of market split over Phase I is small. These crucial magnitudes become larger as our exogenously set value for S_0^L is increased from its current value of 7 units.

Solving this case follows the steps for Case B above exactly but now with a relatively small stock size for L. We set $c^L = 4.9$ and $c^R = 0.901$.[12] We work back in time. For Phase III, we set L's terminal, aggregate supply small, namely 16 units. We invoke rent shrinking at r%. This leaves L with 14.0259 units of 40 (size of G) at the beginning of Phase III. L's time in Phase III is 2.20083 and R's time is 2.6660. R is putting 36.16034 units on the market in Phase III. Now for Phase II. We work back with $Q^L_T = 14.0259$. Working back involves selecting a trial value for L's supply for Phase II and observing how close the corresponding initial supply from R is to G. Repeated trials reveal that this initial supply from R is asymtotically approaching G (=0.005) as L's corresponding supply is increased. For L's supply at 190 units for Phase II, R's $Q^R(t)$ is $G - 0.005000$. For this solution[13] each initial rent is infinitesimal with $\lambda^R(t)$ (=2.82×10^{-432})> $\lambda^L(t)$ (=1.89×10^{-432}). R's supply over this interval is 398,138.0 units.

We arrived at this solution after (a) observing that we needed $c^L - c^R \cong \alpha G$ and (b) re-solving for larger and larger values of L's supply in Phase II. Clearly this solution is the one we expect (or hope for) in a qualitative sense but its asymptotic nature is surprising.

7 Concluding Remark

Kolstad worked with one solution to his model (our Case A above; small high-quality initial stock) and left open how other solutions might work or not work. We have investigated three other types of solution to his model and have turned up three new results. First, other cases do indeed have fairly regular solutions within his framework. Secondly, the "opposite" to his case (small initial low-quality stock) has a well-behaved solution from an economics perspective but displays a somewhat ill-behaved

[12] These costs differ by almost exactly αG. This turns out to matter.

[13] The value $Q^L(t) = 0.0050$ that is being reached depends on our choice of gap size, $[c^R + \alpha G] - c^L$. This has magnitude 0.0010 in our example. The smaller we set this gap, the smaller is the value of $Q^L(t)$ being approached, backwards in time.

solution from a numerical perspective (the solution is asymptotic). Third and most important, we observed regular cases in which the high quality deposit ends up "earning" a lower resource rent. We inferred that relative abundance was the force driving down resoure rent for this case (our Case C).

We observe then that Hotelling extraction theory "goes through" in a simple spatial setting (Kolstad's model) but one new phenomenon obtains, namely the possibility of a high quality deposit "earning" lower resource rent.

References

[1] Adelman, Morris A. and G.C. Watkins (1992) "Reserve asset values and the "hotelling valuation principle"", Working paper (Massachusetts Institute of Technology. Center for Energy Policy Research) ; MIT-CEPR 92-004.

[2] Cairns, Robert D. and Graham A. Davis (2001) "Adelman's Rule and the Petroleum Firm," *The Energy Journal*, vol. 22 (3), pp. 31-54.

[3] Gaudet, Gerard, Michel Moreaux, and Stephen W. Salant (2001) "Intertemporal Depletion of Resource Sites by Spatially Distributed Users", *American Economic Review*, 91, 4, September, pp. 1149-1159.

[4] Herfindahl, Orris C. (1967) "Depletion and Economic Theory" in *Extractive Resources and Taxation*, ed. Mason Gaffney, Madison: University of Wisconsin Press, pp. 63-90.

[5] Hotelling, Harold (1931) "The Economics of Exhaustible Resources", *Journal of Political Economy*, 39, 2, April, pp. 137-175.

[6] Hotelling, Harold (1929) "Stability in Competition", *Economic Journal*, 39, pp. 41-57.

[7] Kolstad, Charles D. (1994) "Hotelling Rents in Hotelling Space: Product Differentiation in Exhaustible Resource Markets", *Journal of Environmental Economics and Management*, 26, pp. 163-180.

[8] Nordhaus, William (1973) "The Allocation of Energy Resources", *Brookings Papers on Economic Activity*, vol. 4, issue 3, pp. 529-576.